EXAM STNM 2009

4 June 2009 - 14:15-16:15

You have 2 hours to complete the exam. All questions have the same value.

1. a. What is the best estimate of the mean of a population given a sample of size N?
   b. The same for the variance.
   c. What is the error of the mean for the same sample?

2. a. What are the mean and variance of the Gaussian distribution?
   b. What are the mean and variance of the Poisson distribution?
   c. Under which condition does the Poisson distribution tend to the Gaussian distribution?

3. Explain the Central Limit Theorem. Why is it such an important theorem?

4. a. Describe how to generate a random number according to any probability distribution function if you know how to generate random numbers uniformly distributed between 0 and 1 using the rejection method.
   b. Consider the distribution described by the equation:

$$P(x) \;=\; \begin{cases} A(1+ax^2) & \text{if } -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   where $P(x) \geq 0$ everywhere within the specified range and the normalizing coinstant A is chosen so that

$$\int_{-1}^{1} P(x)dx = 1.$$

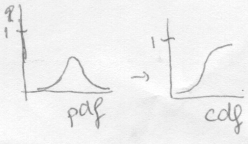
   If you can generate random numbers r uniformly distributed between 0 and 1 in a computer, describe how you can generate random mumbers with pdf $P(x)$ using the inversion method. (Write down the equation that relates x and r; you don't need to solve it.)

5. How would you propagate errors numerically (with a computer program) if you have a function $f(x, y, z)$, and you know the errrors in x, y and z, but the function is too complex to do all the derivatives?

6. If the probability that I am involved in a car accident is 10^{-2} per year, what is the probability that I am invovled in one accident if I drive 30 years? What is the maximum number of years in a row that I can safely drive such that the probability of having an accident is below 50%?

7. a. What is the pdf of the sum of the squares of N random variables, each of them drawn from a Gaussian distribution?
   b. What does this distribution tend to when $N \to \infty$?

$y=ax$ $\quad \sigma_y^2=\left(\frac{\partial x}{\partial y}\right)^2\sigma_x^2$



$f(x+dx,y,z)-f(x,y,z)$

${}^{10}\log 6 = x$ $\quad g^b=a$

$\sum_{i=1}^{N} x_i^2$ $\quad N\to\infty$ $\quad 10^x=6$ $\quad {}^g\log a = b$ $\quad 2^3=8$

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